Intro to Probability
(Day 1)

Definitions:

1) An <u>experiment</u> is a procedure that results in one of a list of outcomes.

Note: You need to be able to repeat an experiment as many times as you wish (repeatable). Each time you perform the experiment, the conditions need to be the same as all other times the experiment is performed.

2) The <u>sample space</u> is a LIST of ALL possible outcomes of an experiment. Notation: S

3) An <u>event</u> is a LIST of SOME possible outcomes of an experiment.

Ex 1 (supp. Hw #1):

Experiment

Flip a single coin once

Events

A = coin lands on heads

B = coin lands on tails

C = coin lands on either heads or tails

D = coin doesn't land on anything

Find

S , A, B, C, D

Hint: One of the 4 events above is the same as S and another one is $\varnothing$

<u>Ex 2 (supp. Hw #5):</u>

Experiment

  Roll a single die once

Events

  A = The die lands on an even number

  B = The die lands on a multiple of 3

  C = The die lands on a prime number

  D = The die lands on 4

  E = The die lands on a number bigger than 2

  F = The die lands on 7

  G = The die lands on a number less than 9

Find

  S , A, B, C, D, E, F, G

Hint: One of the 7 events above is the same as S and another one is $\emptyset$

Probability

Definition:

4) A <u>probability</u> is a number assigned to an event that tells you the chance that the event will occur.

Note:

A probability is a number and is always between 0 and 1 (or 0% and 100%)

If E is an event, its probability is denoted by *P(E)*.

Calculating Probabilities

There are 2 ways to calculate probabilities

I) The classical method

If each outcome of the experiment is equally likely, then

$$P(E) = \frac{|E|}{|S|}$$

<u>Ex 1 (supp. Hw #1, continued):</u>

Experiment
  Flip a single coin once

Events
  A = coin lands on heads
  B = coin lands on tails
  C = coin lands on either heads or tails
  D = coin doesn't land on anything

Find
  S , A, B, C, D, $P(A)$, $P(B)$, $P(C)$, $P(D)$

Hint: One of the 4 events above is the same as S and another one is $\varnothing$

Ex 2 (supp. Hw #5, continued):

Experiment

  Roll a single die once

Events

  A = The die lands on an even number

  B = The die lands on a multiple of 3

  C = The die lands on a prime number

  D = The die lands on 4

  E = The die lands on a number bigger than 2

  F = The die lands on 7

  G = The die lands on a number less than 9

Find

  S , A, B, C, D, E, F, G, $P(A)$, $P(B)$, $P(C)$, $P(D)$, $P(E)$, $P(F)$, $P(G)$, $P(S)$, $P(\emptyset)$

Hint: One of the 7 events above is the same as S and another one is $\emptyset$

<u>Ex 3 (supp. Hw #3):</u>

Experiment

  Flip a single coin 3 times (or flip 3 coins all at the same time, once)

Events

  A = The first and third coin land on the same thing

  B = All coins land on the same thing

  C = The 2nd coin lands on tails

  D = The results of each flip alternate

Find

  S , A, B, C, D, $P(A)$, $P(B)$, $P(C)$, $P(D)$, $P(S)$, $P(\emptyset)$

<u>Ex 4 (supp. Hw #6):</u>

Experiment

  Roll a single die twice (or roll a pair of dice once)

Events

  A = The total of the numbers on the die is 7

  B = The total of the numbers on the die is at least 11

  C = The total of the numbers on the die is at most 4

  D = The absolute value of the difference of the numbers on
      the die is 2

  E = You roll doubles

  F = The first die lands on 5

  G = The first die lands on an even number and the 2nd lands
      on an odd number

Find

  S , A, B, C, D, E, F, G, $P(A)$, $P(B)$, $P(C)$, $P(D)$, $P(E)$, $P(F)$,
$P(G)$, $P(S)$, $P(\emptyset)$

<u>Ex 5 (supp. Hw #9):</u>

Experiment

  Play a single game of roulette

Events

  A = The ball lands in an odd slot

  B = The ball lands in a red slot

  C = The ball lands in a green slot

  D = The ball lands on a number that is a multiple of 3

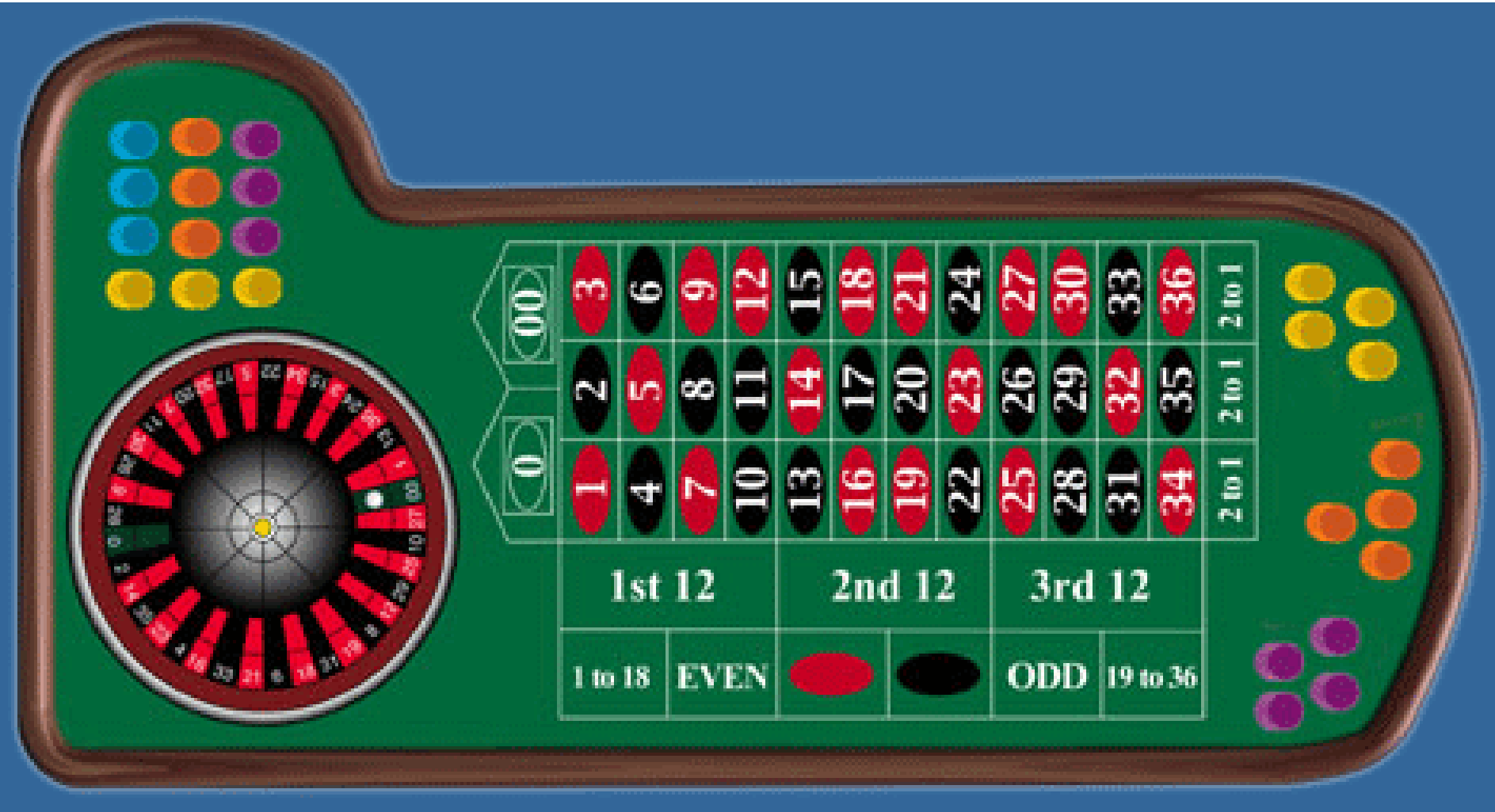 E = The ball lands in a slot that is part of the 1st 12 bet

  F = The ball lands in a slot that is part of the 3rd 12 bet and is
     a black number

  G = The ball lands on a number that is part of the 2nd column

Find

  S , A, B, C, D, E, F, G, $P(A)$, $P(B)$, $P(C)$, $P(D)$, $P(E)$, $P(F)$, $P(G)$, $P(S)$, $P(Ø)$

Ex 6 (supp. Hw #10):

Experiment

  Draw a single card from a standard poker deck

Events

  A = Draw a heart

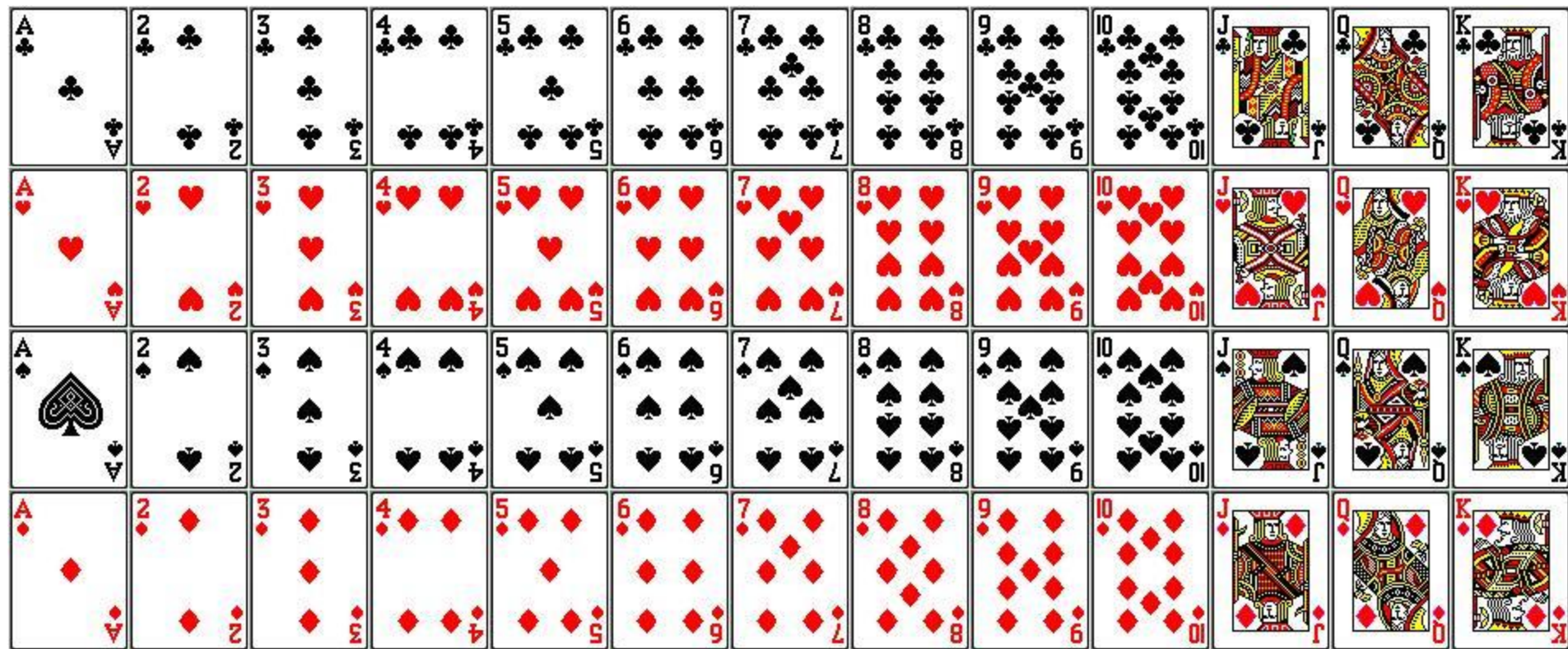 B = Draw a black card

  C = Draw a red face card

  D = Draw a king

  E = Draw a card that has a number on it that is less than 5

Find

  S , A, B, C, D, E, $P(A)$, $P(B)$, $P(C)$, $P(D)$, $P(E)$, $P(S)$, $P(\emptyset)$

Ex 6 (supp. Hw #10): Picture

<u>Ex 7 (supp. Hw #13):</u>

13)

Experiment

  Draw a single ball from the bag pictured below

Events

  A = You draw a ball with the number 3 on it

  B = You draw a ball with the number 7 on it
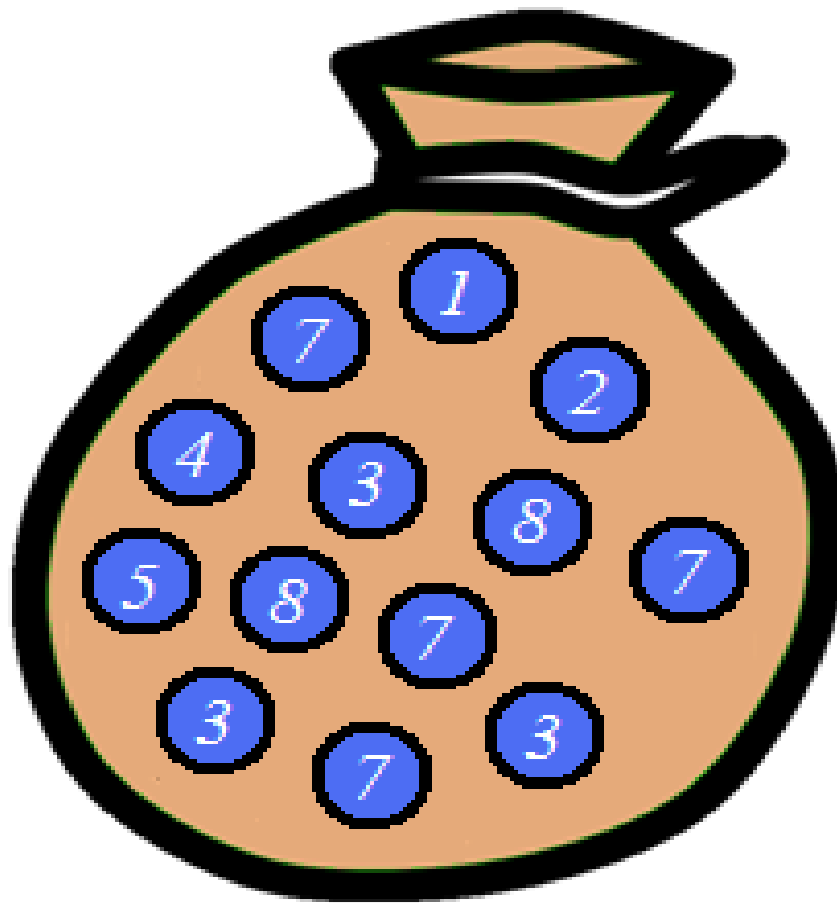
  C = You draw a ball with the number 5 on it

  D = You draw a ball that has an even number on it that is also larger than 3

  E = You draw a ball that has a prime number on it

Find

  S , *A*, *B*, *C*, *D*, *E*, *P(A)*, *P(B)*, *P(C)*, *P(D)*, *P(E)*, *P(S)*, *P(Ø)*

Picture

Calculating Probabilities

There are 2 ways to calculate probabilities

II) The empirical method

Run the experiment many times, then …

$$P(E) = \frac{\text{\# of times the event occured}}{\text{\# of times you ran the experiment}}$$

$P(E) =$ The percentage of times the event occurred out of all runs of the experiment

Calculating Empirical Probabilities
In Class Project
Find *P(E)* Empirically

1) Experiment = Draw a single chip from a bag
   Event E = You draw a blue chip

2) Experiment = Draw 2 cards from a deck without replacement
   Event E = The total of the 2 cards is 20

3) Experiment = Roll a pair of dice
   Event E = The total of the numbers on the dice is 7